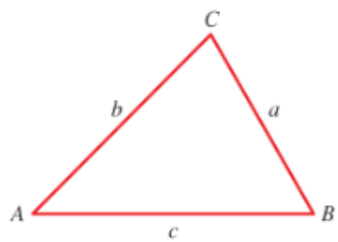
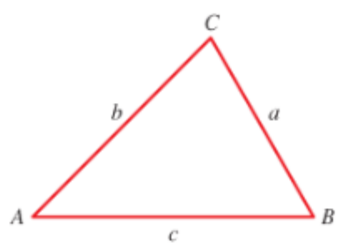
Unit 4– Law of Sines & Cosines, Identities and Equations

6.5 Law of Sines

The next two sections discuss how we can "solve" (find missing parts) of _________________(non-right) triangles.



If we create right triangles by dropping a perpendicular from C to the side AB, we can use what we know about right triangles to find parts of triangle ABC.

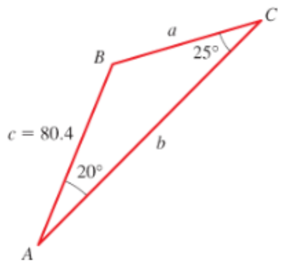


Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ which can also be written. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

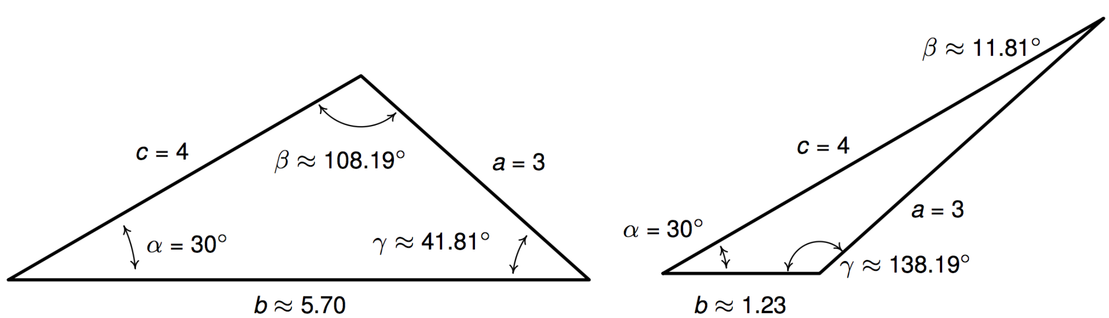
Law of Sines Examples

Example 2 (book): Find the remaining parts:

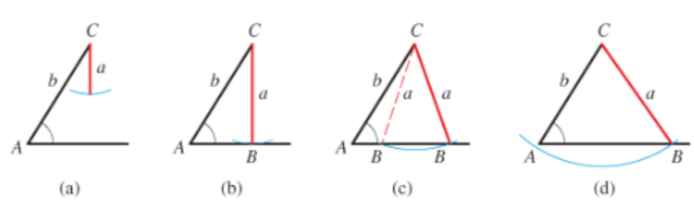


Example: $A = 30^\circ,\ a = 1,\quad c = 4$

Example: $A = 30^\circ,\ a = 3,\quad c = 4$

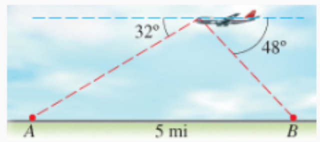


The ambiguous case



Applications

32. **Flight of a Plane** A pilot is flying over a straight highway. He determines the angles of depression to two mileposts, 5 mi apart, to be $32°$ and $48°$, as shown in the figure.

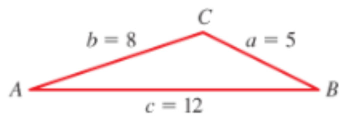


(a) Find the distance of the plane from point A.

(b) Find the elevation of the plane.

6.5 Law of Cosines

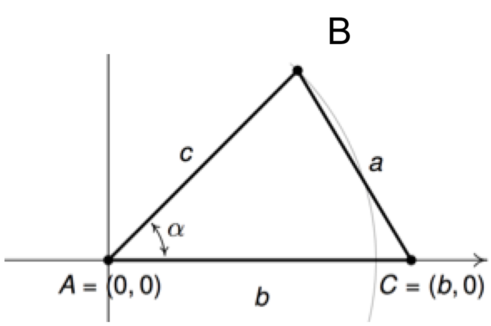
Example: Use the Law of Sines to find the remaining parts of the triangle shown



Development of the Law of Cosines

Can we find a relationship relating the sides of an oblique triangle? Suppose we superimpose a coordinate system onto a general triangle as shown.

What are the coordinates of point B? ____________________________

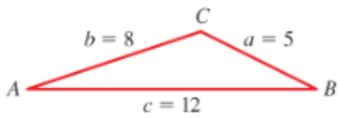


Law of Cosines

$$a^2 = b^2 + c^2 - 2ab\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

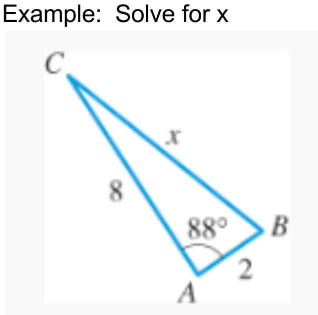
$$c^2 = \_\_\_\_^2 + \_\_\_\_^2 - 2\_\_\_\ \_\_\_\cos\_\_\_\_\_$$

Example: Find all the remaining parts



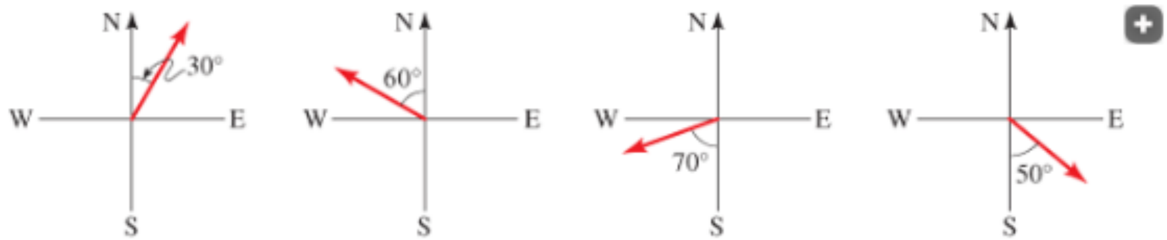
Tip: It is helpful to find the ___________________________first. (If we find the largest angle first, the others must be acute)

Example: Solve for x



Example:
Two cars leave a city at the same time and travel along straight highways that differ in direction by 128°. If their speeds are 60mph and 50mph respectively, how far apart are the cars at the *ans: 33 miles*

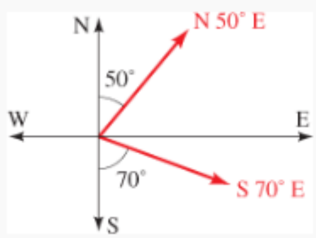
Navigation (Bearing).



N 30° E   N 60° W   S 70° W   S 50° E

Example:

44. Navigation Two boats leave the same port at the same time. One travels at a speed of 30 mi/h in the direction N 50° E, and the other travels at a speed of 26 mi/h in a direction S 70° E (see the figure). How far apart are the two boats after 1 h?



7.1 Simplifying Trigonometric Expressions and Basic Identities.

Simplifying Trig. Expressions - Examples

1) $\tan\theta\csc\theta$

2) $\dfrac{\cos x\sec x}{\cot x}$

3) $\sin^3 t + \sin t\cos^2 t$

4). $\dfrac{\sin x}{1-\cos x} - \csc x$

Proving Identities

Condition Equation vs. Identities

To prove identities, you need to start with one (either) side, and show step-by-step how to get to the other. Presentation is really important. In the end you must have the left side connected to the right side by "=" signs, being careful not to write "=" until you have shown it.

Recall the Pythorgean Identities and all the different forms they can take.

$\cos^2 x + \sin^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
$1 - \cos^2 x = \sin^2 x$	$\tan^2 x = \sec^2 x - 1$	$\cot^2 x = \csc^2 x - 1$
$1 - \sin^2 x = \cos^2 x$	$\sec^2 x - \tan^2 x = 1$	$\csc^2 x - \cot^2 x = 1$

You will use the simplifying techniques we just practiced along with the identities we have learned thus far.

Examples and Presentation Tips

DO

1) $\tan\theta\cos\theta = \sin\theta$

2) $2\cos^2 x - 1 = 1 - 2\sin^2 x$

DON'T

1) $\tan\theta\cos\theta = \sin\theta$

2) $2\cos^2 x - 1 = 1 - 2\sin^2 x$

It is best to start with the more complicated side.

3). $1 + \cos^2 t = \dfrac{1 + \sec^2 t}{1 + \tan^2 t}$

Sometimes putting everything in terms of sine and cosine helps.

4). $1+\cos^2 t=\dfrac{1+\sec^2 t}{1+\tan^2 t}$

If necessary, you can work on both side separately until they match, but you have to be especially careful with presentation in this case.

5). $\csc y-\sin y=\cos y\ \cot y$

7.4 More Solving Trigonometric Equations

(we are covering this out of order, we will return to 7.2 and 7.3 after this)

Now that we can manipulate the trig functions, we can solve more types of trigonometric equations. Here, we will use factoring and the Zero Factor Law.

1). $2\sin^2 t-11\sin t+5=0$

2) $3\sin x\cos x=2\sin x$

Why don't we just divide by $\sin x$?

7.2 Sum and Difference Formulas

Recall Function Notation: Does $\cos\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{3}\right)+\cos\left(\frac{\pi}{6}\right)$?________________

In general, . $f(x+y)$__________ $f(x)+f(y)$

Likewise, for the trig functions

$\cos(a+b)$__________ $\cos(a)+\cos(b)$

Here are the addition and subtraction identities for sine, cosine, and tangent.

See the derivation in the book.

$$\sin(a+b)=\sin a\cos b+\cos a\sin b$$
$$\sin(a-b)=\sin a\cos b-\cos a\sin b$$
$$\cos(a+b)=\cos a\cos b-\sin a\sin b$$
$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$
$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}$$
$$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

Using the sum and difference formulas:

Ex: Find the exact value of

(a). $\sin(15^\circ)$

(b). $\tan\left(\frac{7\pi}{12}\right)$

Ex: Using the identities *backwards* – Find the exact value $\cos 40^\circ\cos 10^\circ+\sin 40^\circ\sin 10^\circ$

Ex: Given that

$$\cos\theta = -\frac{1}{3}, \quad \theta \text{ in Quadrant II}$$
$$\tan\phi = 2, \quad \phi \text{ in Quadrant III}$$

Find. $\cos(\theta+\phi)$ exactly.

Ex: Find $\sin\left(\sin^{-1}\left(-\frac{1}{4}\right) - \cos^{-1}\left(\frac{2}{3}\right)\right)$ exactly

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Prove the Identity

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

(Npte: There are some additional topics that we do not cover in the book)

7.2 Double Angle, Half Angle, Power Reducing and Product-Sum Formulas

Double Angle Formulas

Recall Function Notation: Does $\sin\left(2\cdot\frac{\pi}{6}\right)=2\sin\left(\frac{\pi}{6}\right)$?_______________

In general, . $f(cx)$__________ $cf(x)$

Likewise, for the trig functions

$\cos(n\theta)$__________ $n\cos(\theta)$

So can anything be done with $\sin(2\theta)$?

$\sin(2\theta)$

Similarly

$\cos(2\theta)$

Double Angle Formulas:

- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 2\cos^2(\theta) - 1 \\ 1 - 2\sin^2(\theta) \end{cases}$
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\tan(2\theta) = \dfrac{2\tan(\theta)}{1-\tan^2(\theta)}$

We use these "frontwards" and "backwards" and as templates:

(a). $\sin(2\theta)=2\sin\theta\cos\theta$

$\Rightarrow \quad \sin\theta\cos\theta=\frac{1}{2}\sin(2\theta)$

So. $\sin 15^\circ \cos 15^\circ =$__________

(b) $\cos(2\theta)=2\cos^2\theta-1$

$\cos($_________$)=2\cos^2$__________-1

So. $\cos(6x)=$___________________

Using the double angle formulas:

Example: Given that P(-1,2) lies on the terminal side of θ, find $\sin 2\theta$.

Example: $\cos\left(2\tan^{-1}\frac{12}{5}\right)$

Example: Find an identity for in terms of θ for $\sin 3x$

EX: Verify the identity $\frac{1-\cos 2x}{\sin 2x} = \tan x$

Power Reducing and Half Angle Formulas

From the Double Angle Formulas for cosine, we can derive other useful identities.

$\cos(2\theta) = 2\cos^2\theta - 1$	$\cos(2\theta) = 1 - 2\sin^2\theta$

Power Reducing Formulas

Half Angle Formulas

Choosing the + or – will depend on the quadrant of. ____________-

Using the power reducing formulas:

Example: Write in terms of terms having power of at most 1. (This is a process that will be very useful in calculus)

$$\cos^4\theta$$

Using the half angle formulas:

Example: Find the exact value of $\sin(15^\circ)$ *(done in the previous section).*

Example: If $\cot x = 5; \quad 180^\circ < x < 270^\circ$, find $\sin\left(\frac{\theta}{2}\right)$

Product to Sum Formulas *(memorization not required)*

Derivation in book

$$\begin{aligned}
\sin u \cos v &= \tfrac{1}{2}[\sin(u+v) + \sin(u-v)] \\
\cos u \sin v &= \tfrac{1}{2}[\sin(u+v) - \sin(u-v)] \\
\cos u \cos v &= \tfrac{1}{2}[\cos(u+v) + \cos(u-v)] \\
\sin u \sin v &= \tfrac{1}{2}[\cos(u-v) - \cos(u+v)]
\end{aligned}$$

Example: Write as a sum: $\sin 3x \cos 7x$

Sum to Product Formulas (also called factoring formulas) *(memorization not required)*

$$\begin{aligned}
\sin x + \sin y &= 2\sin\tfrac{x+y}{2}\cos\tfrac{x-y}{2} \\
\sin x - \sin y &= 2\cos\tfrac{x+y}{2}\sin\tfrac{x-y}{2} \\
\cos x + \cos y &= 2\cos\tfrac{x+y}{2}\cos\tfrac{x-y}{2} \\
\cos x - \cos y &= -2\sin\tfrac{x+y}{2}\sin\tfrac{x-y}{2}
\end{aligned}$$

Example: Express as a product: $\sin 5x - \sin 3x$

7.5 Solving Trigonometric Equations Using New Identities

1). Solve $3\sin 2\theta - 2\sin\theta = 0 \qquad 0 \le \theta \le 2\pi$

2) Solve $2\cos 2\theta - 1 = 0 \qquad 0 \le \theta \le 2\pi$

3). Solve $2\cos^2 x - \sin x - 1 = 0$

4) $\cos\theta - \sin\theta = 1 \quad 0 \le \theta \le 2\pi$

5). $\cos 5\theta - \cos\theta = 1 \quad 0 \le \theta \le 2\pi$